Azimuth and Amplitude

Azimuth

The azimuth of a heavenly body is the angle at the observer's zenith contained between the observer's meridian and the vertical circle passing through the body. Basically, azimuth is the bearing of the heavenly body with respect to the observer. The azimuth can be calculated either mathematically, or by using the Azimuth Tables. The azimuth is used to observe the compass error by taking the bearing of the heavenly body and comparing it with the result of the azimuth calculation.

Procedure for obtaining the azimuth and compass error

1. Ascertain the UT to conduct the azimuth; if necessary, convert time to LMT. This will make later calculation easier, and also provide the approximate bearing of the body in advance.

2. From the Nautical Almanac, extract the GHA and the declination of the body. For the star, SHA of the star and GHA for the first point of Aries are needed. Then calculate LHA.

3. With DR position, calculate the azimuth by ABC table, or by using ABC formula as follows:

$$\mathbf{A = \frac{\tan Lat.}{\tan LHA}}$$ A is named opposite to latitude unless LHA is between 90° and 270°

$$\mathbf{B = \frac{\tan Dec.}{\sin LHA}}$$ B is named same as declination

$$\mathbf{C = A \pm B}$$ C is named as A or B, whichever is greater

$$\mathbf{Azimuth = \tan^{-1}\left(\frac{1}{C \times \cos Lat.}\right)}$$

Azimuth is expressed in quadrantal notation and in the form:
N or S Azimuth E or W

N or S: same as C

E or W: W if LHA < 180°
E if LHA > 180°

Azimuth also can be found by following formula:

$$\mathbf{AZ = \tan^{-1}\left(\frac{\sin LHA}{\tan Dec. \cos Lat. - \cos LHA \sin Lat.}\right)}$$

(South value is entered as negative value)

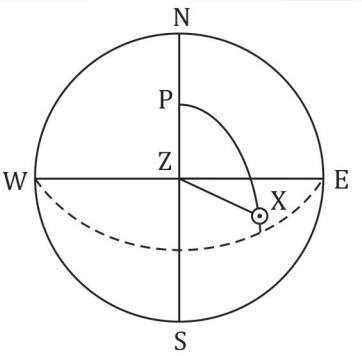
- If denominator is negative, azimuth will be named South (S).

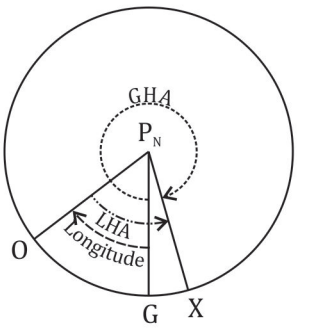
- If denominator is positive, azimuth will be named North (N).
- If LHA is between 0° and 180°, azimuth will be named West (W).
- If LHA is between 180° and 360°, azimuth will be named East (E).

4. Convert azimuth from quadrantal notation form to three-figure notation form, then compare the azimuth with the compass reading to figure out the compass error by the following rules.

Compass **BEST**, Compass Error **WEST**
Compass **LEAST**, Compass Error **EAST**

Example 1 On 17th April 2008, at about 0800, DR position 47°30′N. 052°30′W., chronometer showed $11^h15^m52^s$, 2^m20^s slow. The sun was observed bearing 135° C, variation 20°W. Calculate the deviation of the compass:





Approx. LMT	17th	$08^h00^m00^s$
Long. (W)		$3^h30^m00^s$
Approx. UT	17th	$11^h30^m00^s$

Chronometer	17th	$11^h15^m52^s$
Error		2^m20^s
UT	17th	$11^h18^m12^s$
Long. (W)		$3^h30^m00^s$
LMT		$7^h48^m12^s$

GHA at 17th $11^h00^m00^s$	345°07.6′
Increments (18^m12^s)	4°33.0′
GHA at 17th $11^h18^m12^s$	349°40.6′
Longitude (W)	52°30.0′
LHA	297°10.6′

Declination	10°41.7′N
d = 0.9	0.3′
Declination	10°42.0′N

Using ABC tables

A	0.56 S	Azimuth :	S076.8°E
B	0.21 N	True bearing:	103.2°T
C	0.35 S		

Using other formula

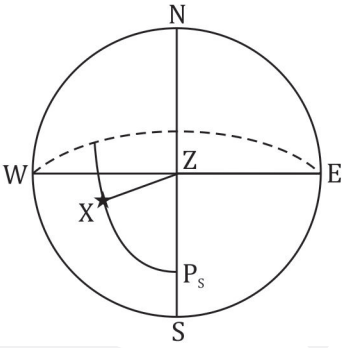
$$AZ = \tan^{-1}\left(\frac{\sin LHA}{\tan Dec.\cos Lat. - \cos LHA \sin Lat.}\right)$$

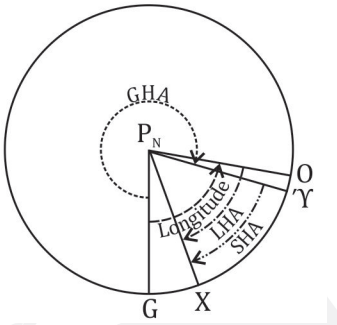
$$= \tan^{-1}\left(\frac{\sin 297°10.6'}{\tan 10°42'\cos 47°30' - \cos 297°10.6'\sin 47°30'}\right)$$

$$= S76.8°E = 103.2°T$$

True Bearing	103.2°T	Compass Error	31.8°W
Compass Bearing	135.0°C	Variation	20°W
Compass Error	31.8°W	Deviation	11.8°W

Example 2 On 19th July 2008, at about 0430, DR position 32°30′S 080°15′E, chronometer showed $10^h15^m20^s$ with no error. Star Peacock was observed bearing 250°C, variation 28°W. Find the compass error and deviation:





Approx. LMT	19th 04h30m	Chronometer	19th 11h15m20s
Long. (E)	5h21m		12h00m00s
Approx. UT	18th 23h09m	UT	18th 23h15m20s

GHA at $18^d\ 23^h\ 00^m\ 00^s$	282°07.3′	Declination	56°42.4′S
Increments ($15^m\ 20^s$)	3°50.6′		
GHA^i at $18^d\ 23^h\ 15^m\ 20^s$	285°57.9′		
SHA	53°24.2′		
GHA	339°22.1′		
Longitude (E)	80°15.0′		
	419°37.1′		
	360°00.0′		
LHA	59°37.1′		

Using ABC formula

$$A = \frac{\tan \text{Lat.}}{\tan \text{LHA}} = \frac{\tan 32°30'}{\tan 59°37.1'} = 0.37\text{N}$$

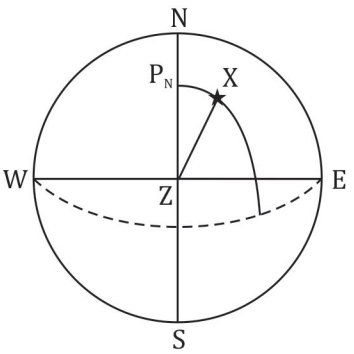
$$B = \frac{\tan \text{Dec.}}{\sin \text{LHA}} = \frac{\tan 56°54.2'}{\sin 59°37.1'} = 1.77\text{ S}$$

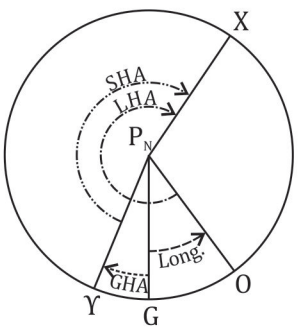
A	0.37 N
B	1.77 S
C	1.40 S

$$\text{Azimuth} = \tan^{-1}\left(\frac{1}{\text{C} \times \cos \text{Lat.}}\right) = \tan^{-1}\left(\frac{1}{1.40 \times \cos 32°30'}\right)$$
$$= \text{S}040.3°\text{W} = 220.3°\text{T}$$

True Bearing	220.3°T	Compass Error	29.7°W
Compass Bearing	250.0°C	Variation	28.0°W
Compass Error	29.7°W	Deviation	1.7°W

Example 3 On 26th October 2008, at approximately 0120 in DR position 34°45′N, 35°15′E, Star Dhube bore 026° by compass. Variation 4° E. Chronometer $10^h55^m42^s$ has error 3^m15^s slow. Find compass deviation:





Approx. LMT	26^{th}	$1^h 20^m$	Chronometer	$10^h 55^m 42^s$	
Long. (E)		$2^h 21^m$	Error	$3^m 15^s$	
Approx. UT	25^{th}	$22^h 59^m$	Chronometer	$10^h 58^m 57^s$	
				$12^h 00^m 00^s$	
			UT	25^{th}	$22^h 58^m 57^s$

GHA ♈ at	25^{th} $22^h 00^m 00^s$	4°39.5′	Declination	61°42.0′N
Increments	($58^m 57^s$)	14°46.7′		
GHA ♈ at	25^{th} $22^h 58^m 57^s$	19°26.2′		
	SHA*	193°56.1′		
	GHA*	213°22.3′		
	Longitude (E)	35°15.0′		
	LHA	248°37.3′		

Find azimuth by using formula:

$$AZ = \tan^{-1}\left(\frac{\sin LHA}{\tan Dec.\cos Lat. - \cos LHA \sin Lat.}\right)$$

$$= \tan^{-1}\left(\frac{\sin 248°37.3'}{\tan 61°42.0'\cos 34°45' - \cos 248°37.3'\sin 34°45'}\right)$$

$$= N28.2°E$$

$$= 028.2°T$$

Find azimuth by using ABC formula:

$$A = \frac{\tan Lat.}{\tan LHA} = \frac{\tan 34°45'}{\tan 248°37.3'} = 0.27\,N$$

$$B = \frac{\tan Dec.}{\sin LHA} = \frac{\tan 61°42.0'}{\sin 248°37.3'} = 1.99\,N$$

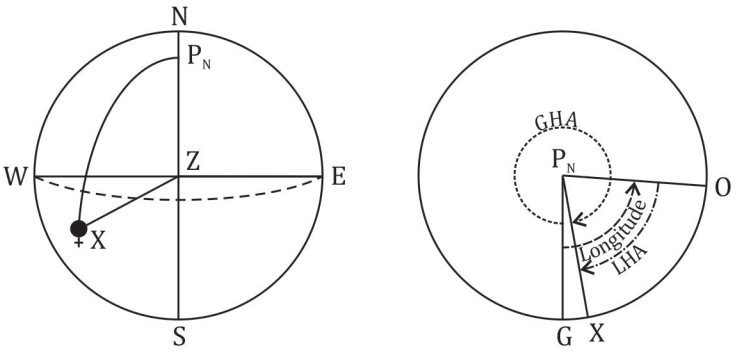
A	0.27 N
B	1.99 N
C	2.26 N

$$Azimuth = \tan^{-1}\left(\frac{1}{C \times \cos Lat.}\right) = \tan^{-1}\left(\frac{1}{2.26 \times \cos 34°45'}\right)$$

$$= N28.3°E = 028.3°T$$

True Bearing	028.3°T	Compass Error	2.3°E
Compass Bearing	026°C	Variation	4°E
Compass Error	2.3°E	Deviation	1.7°W

Example 4 At approximately 19:40 on 26th October 2008, the chronometer showed $1^h42^m25^s$; chronometer error is nil. DR position 14°18′N, 88°18′E; observed Venus bore 246° compass, variation 2° W. Find compass deviation:



Approx. LMT	18th 19^h40^m	Chronometer	$1^h42^m25^s$
Long. (E)	5^h53^m	Error	nil
Approx. UT	18th 13^h47^m	Chronometer	$1^h42^m25^s$
			$12^h00^m00^s$
		UT	18th $13^h42^m25^s$

GHA	342°46.7′	Declination	23°16.7′ S
Increments	10°36.3′	d = 0.6′	0.4′
GHA	353°23.0′		23°17.1′ S
v = −0.8′	−0.6′		
GHA	353°22.4′		
Long. (E)	88°18′		
	441°40.4′		
	360°		
LHA	81°40.4°		

$$A = \frac{\tan \text{Lat.}}{\tan \text{LHA}} = \frac{\tan 14°18'}{\tan 81°40.4'} = 0.04S$$

$$B = \frac{\tan \text{Dec.}}{\sin \text{LHA}} = \frac{\tan 23°17.1'}{\sin 81°40.4'} = 0.44S$$

A	0.04 S
B	0.44 S
C	0.48 S

$$AZ = \tan^{-1}\left(\frac{1}{C \times \cos \text{Lat}}\right) = \tan^{-1}\left(\frac{1}{0.48 \times \cos 14°18'}\right)$$

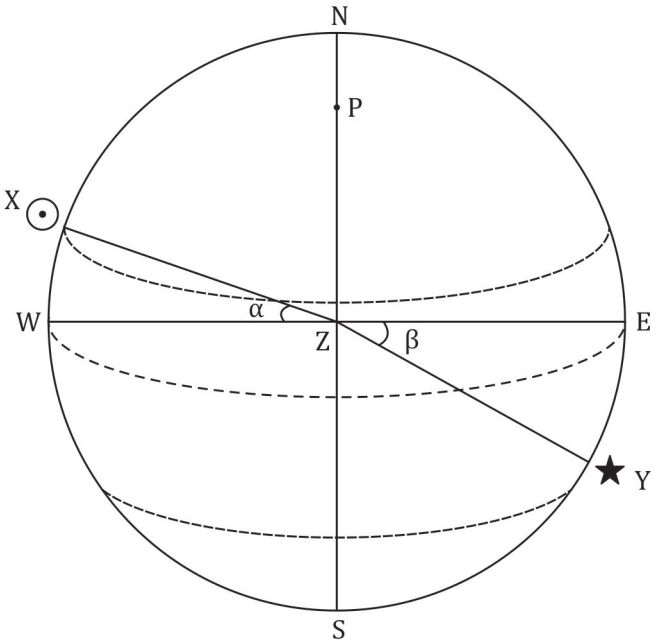
$$= S65.1°W$$

$$= 245.1°T$$

True bearing	245.1°T	Compass error	0.9°W
Compass bearing	246.0°C	Variation	2°W
Compass error	0.9°W	Deviation	1.1°E

Amplitude

The amplitude of a heavenly body is the arc of horizon contained between the position of the body when rising or setting and the East or West point of the horizon. Alternatively, it is the angle between the bearing of the body when rising or setting and the East or West direction.



In the above diagram, α° is the amplitude of body X, which is setting and has north declination, so the amplitude is named as Wα°N. β° is the amplitude of body Y, which is rising and has south declination, so it is named Eβ°S. Basically, the amplitude of a heavenly body is named East if rising, and West if setting, followed by North or South depending on the declination of the body at time of observing.

$$\textbf{Amplitude} = \frac{\textbf{sinDeclination}}{\textbf{cosLatitude}}$$

The amplitude is a simple and quick method for checking compass error. The bearing of a heavenly body when rising or setting is calculated in advance, which is the true bearing. At the moment it is at the horizon, the bearing is taken with the compass. Compare the true bearing and compass bearing to establish the compass error.

Example 5 On 18th July 2008, in position 50°30′N, 20°15′W, the sun rose bearing 080° by compass, variation 10° W. Find compass error and compass deviation:

LMT 50°N	$18^d\ 04^h11^m$	Declination	20°57.5′N
LMT 52°N	$18^d\ 04^h02^m$	d = 0.4′	−0.2′
Difference	9^m	Declination	20°57.3′N
Latitude Correction	-2^m	(table I)	
LMT 50°30′N	$18^d\ 04^h09^m$		
Longitude in Time	1^h21^m	(20°15′W)	
UT	$18^d\ 05^h30^m$		

$$\text{Amplitude} = \sin^{-1}\left(\frac{\sin \text{Dec.}}{\cos \text{Lat.}}\right) = \sin^{-1}\left(\frac{\sin 20°57.3'}{\cos 50°30.0'}\right) = \text{E}34.2°\text{N}$$

True Bearing	055.7°T	Compass Error	24.2°W
Compass Bearing	080.0°C	Variation	10.0°W
Compass Error	24.2°W	Deviation	14.2°W

Example 6 On 17th April 2008, in position 48°45′S, 158°05′E, the sun rose bearing 067° by compass, variation 24° E. Find compass error and compass deviation:

LMT 45°S	$17^d\ 06^h37^m$	Declination	10°28.5′N
LMT 50°S	$17^d\ 06^h44^m$	d = 0.9′	+0.2′
Difference	7^m	Declination	10°28.7′N
Latitude Correction	$+5^m$	(table I)	
LMT 48°45′S	$17^d\ 06^h42^m$		
Longitude in Time	10^h32^m	(158°05′E)	
UT	$16^d\ 20^h10^m$		

$$\text{Amplitude} = \sin^{-1}\left(\frac{\sin \text{Declination}}{\cos \text{Latitude}}\right) = \sin^{-1}\left(\frac{\sin 10°28.7'}{\cos 48°45.0'}\right) = \text{E}\ 16°\text{N}$$

True Bearing	074°T	Compass Error	7°E
Compass Bearing	067°C	Variation	24°E
Compass Error	7°E	Deviation	17°W

Example 7 On 25th October 2008, in position 30°45′S, 160°15′E, the sun set bearing 250° by compass, variation 14° E. Find deviation for the ship's head:

LMT Sunrise 30°S	$25^d\ 18^h17^m$		Declination	12°15.0′S
LMT Sunrise 35°S	$25^d\ 18^h24^m$		d = 0.9′	+0.6′
Difference		7^m	Declination	12°15.6′S
Latitude Correction		$+1^m$ (table I)		
LMT Sunrise 30°45′S	$25^d\ 18^h18^m$			
Longitude in Time		10^h41^m (160°15′E)		
UT Sunrise	$25^d\ 07^h37^m$			

$$\text{Amplitude} = \sin^{-1}\left(\frac{\sin \text{Dec.}}{\cos \text{Lat.}}\right) = \sin^{-1}\left(\frac{\sin 12°15.6'}{\cos 30°45.0'}\right) = \text{W } 14.3°\text{S}$$

True Bearing	255.7°T	Compass Error	5.7°E
Compass Bearing	250.0°C	Variation	14.0°E
Compass Error	5.7°E	Deviation	8.3°W